Large cliques in GIRGs

Master Thesis - Project Description

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Geometric Inhomogeneous Random Graphs (GIRGs) are a versatile, weighted geometric model for real-world networks, where each vertex draws a weight from a given distribution and a position uniformly at random in a geometric ground space [3]. With these data fixed, pairs of vertices are connected independently of other pairs. The connection probability increases with the product of the vertex weights of the pair and is inversely proportional to a power of its geometric distance. In particular, the following hold for GIRGs [2]:

- GIRGs are sparse (they contain $\Theta(n)$ edges) and the vertex degrees follow a power-law.
- GIRGs have a unique giant component containing $\Theta(n)$ vertices.
- GIRGs have polylogarithmic diameter, with $O(\log\log(n))$ average distance in the giant component.
- GIRGs have constant clustering coefficient.

The natural next step to further study clustering properties is to examine cliques of sizes larger than 3. In [4] the number of cliques of any constant size k contained in a GIRG is determined precisely, and the typical clique is described. The goal of this thesis is to investigate cliques of size $k = \omega(1)$ growing with the number of vertices n. For example, the size of the largest clique in a GIRG is a quantity of interest. It has been determined for Hyperbolic Random Graphs [1], of which GIRGs are a generalization, but is not known in general.

Goal of the project Over the course of this thesis, you will investigate cliques of super-constant size in GIRGs. In particular, you will

- Get acquainted with the current state of research on GIRGs, in particular about cliques.
- Study the number of cliques of size $k = \omega(1)$ in a GIRG, and the size of the largest clique in a GIRG.
- Attempt to describe the typical cliques of super-constant size.

More information and a grading scheme can be found at: https://www.cadmo.ethz.ch/education/thesis/guidelines.html

Prerequisites Random graphs. 'Randomised Algorithms and Probabilistic Methods' is helpful but not strictly necessary.

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Contact Please contact Marc and Ulysse if you are interested in the project, and tell us a little bit about your mathematical background (e.g. attach a list of courses taken or a transcript of records).

References

[1] Thomas Bläsius, Tobias Friedrich, and Anton Krohmer. "Cliques in hyperbolic random graphs". In: *Algorithmica* 80.8 (2018), pp. 2324–2344.

[2] Karl Bringmann, Ralph Keusch, and Johannes Lengler. "Average distance in a general class of scale-free graphs with underlying geometry". In: *arXiv preprint arXiv:1602.05712* (2016).

[3] Karl Bringmann, Ralph Keusch, and Johannes Lengler. "Geometric Inhomogeneous Random Graphs". In: *Theoretical Computer Science* 760 (2015).

[4] Riccardo Michielan and Clara Stegehuis. "Cliques in geometric inhomogeneous random graphs". In: *Journal of Complex Networks* 10.1 (2022).