Expressivity of GIRGs

Master Thesis - Project Description

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Geometric Inhomogeneous Random Graphs (GIRGs) are a versatile, weighted geometric model for real-world networks, where each vertex draws a weight from a given distribution and a position uniformly at random in a geometric ground space [4]. With these data fixed, pairs of vertices are connected independently of other pairs. The connection probability increases with the product of the vertex weights of the pair and is inversely proportional to a power of its geometric distance. In particular, the following hold for GIRGs [3]:

- GIRGs are sparse (they contain $\Theta(n)$ edges) and the vertex degrees follow a power-law.
- GIRGs have a unique giant component containing $\Theta(n)$ vertices.
- GIRGs have polylogarithmic diameter, with $O(\log\log(n))$ average distance in the giant component.
- GIRGs have constant clustering coefficient.

A central question of random graph models concerns their *expressivity* [1], [2], i.e. how well the features of real-world networks can be matched by tuning the parameters of the random graph models. In this thesis, you will examine how increasing the dimension of a GIRG may improve such a fit.

Goal of the project Over the course of this thesis, you will

- Get acquainted with the current state of research on GIRGs.
- Analyse what graph properties characterize real-world networks
- Investigate how the dimension of the feature space affects the fit quality of GIRGs to real-world networks.

More information and a grading scheme can be found at: https://www.cadmo.ethz.ch/education/thesis/guidelines.html

Prerequisites Random graphs. 'Randomised Algorithms and Probabilistic Methods' and 'Algorithms Lab' are helpful but not necessary. Familiarity with tools for network analysis such as NetworKit may be beneficial.

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Contact Please contact Marc and Ulysse if you are interested in the project, and tell us a little bit about your mathematical background (e.g. attach a list of courses taken or a transcript of records).

References

[1] Thomas Bläsius, Tobias Friedrich, and Maximilian Katzmann. "Force-Directed Embedding of Scale-Free Networks in the Hyperbolic Plane". In: *19th International Symposium on Experimental Algorithms (SEA 2021)*. Ed. by David Coudert and Emanuele Natale. Vol. 190. Leibniz International Proceedings in Informatics (LIPIcs). Dagstuhl, Germany: Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2021, 22:1–22:18. ISBN: 978-3-95977-185-6. DOI: 10.4230/LIPIcs.SEA.2021.22. URL: https://drops.dagstuhl.de/opus/volltexte/2021/13794.

[2] Thomas Bläsius et al. "Towards a systematic evaluation of generative network models." In: *International Workshop on Algorithms and Models for the Web-Graph* (2018), pp. 99–114.

[3] Karl Bringmann, Ralph Keusch, and Johannes Lengler. "Average distance in a general class of scale-free graphs with underlying geometry". In: *arXiv preprint arXiv:1602.05712* (2016).

[4] Karl Bringmann, Ralph Keusch, and Johannes Lengler. "Geometric Inhomogeneous Random Graphs". In: *Theoretical Computer Science* 760 (2015).