Size of separators in non-metric GIRGs

Master Thesis - Project Description

August 2022

Geometric Inhomogeneous Random Graphs (GIRGs) are a versatile, weighted geometric model for real-world networks, where each vertex draws a weight from a given distribution and a position uniformly at random in a geometric ground space [2]. With these data fixed, pairs of vertices are connected independently of other pairs. The connection probability increases with the product of the vertex weights of the pair and is inversely proportional to a power of its geometric distance. Informally, two vertices are more likely connected, if they are "similar" to each other, i.e. geometrically close. In particular, the following hold for GIRGs [1]:

- GIRGs are sparse (they contain $\Theta(n)$ edges) and the vertex degrees follow a power-law.
- GIRGs have a unique giant component containing $\Theta(n)$ vertices.
- GIRGs have polylogarithmic diameter, with $O(\log\log(n))$ average distance in the giant component.
- GIRGs have constant clustering coefficient.

The ground space from which the positions of the vertices are drawn is the d-dimensional unit cube $[0,1]^d$, usually equipped with the max-distance $\|x-y\|_\infty := \max_{i\in[d]} |x_i - y_i|$. In this case, there exist separators of small size, which means that it is possible to split its giant component into two parts of linear size each by deleting $O(n^{1-\varepsilon})$ edges for some small constant $\varepsilon > 0$ [2]. Another choice of distance that gives rise to an interesting geometry is the *minimum component distance* $\|x-y\|_{\min} := \min_{i\in[d]} |x_i - y_i|$. This is motivated by the following intuition from social networks: For two people to know each other, it suffices that they are similar with regard to one of their characteristics. In this case, assuming $d > 1$, the picture for separator sizes is different, namely no small separator exists, which means that any set of edges whose removal splits the giant component in two parts of linear size must contain $\Theta(n)$ edges [3]. The goal of this thesis is to investigate how the size of separators behaves for choices of distances that interpolate between the Euclidean distance $\|\cdot\|_\infty$ and the minimum component distance $\|\cdot\|_{\min}$.

Goal of the project Over the course of this thesis, you will investigate how the choice of the underlying geometry influences the existence of small separators in GIRGs. In particular, you will

- Get acquainted with the current state of research on GIRGs, including the proof methods related to size of separators in [2] and [3].
- Examine the size of separators for various other choices of distance. A natural place to start could be a mixture of $\|\cdot\|_\infty$ and $\|\cdot\|_{\min}$ in dimension $d = 3$, with the following two distances:

$$\|x-y\|_\wedge := \min\{|x_1-y_1|, \max\{|x_2-y_2|, |x_3-y_3|\}\},$$
$$\|x-y\|_\vee := \max\{|x_1-y_1|, \min\{|x_2-y_2|, |x_3-y_3|\}\}.$$

More information and a grading scheme can be found at: https://www.cadmo.ethz.ch/education/thesis/guidelines.html

Prerequisites Random graphs. 'Randomised Algorithms and Probabilistic Methods' is helpful but not strictly necessary.

Supervisors Marc Kaufmann (kamarc@ethz.ch) & Ulysse Schaller (ulysses@ethz.ch), CAB G 36.2

Supervising professor Prof. Dr. Angelika Steger (steger@inf.ethz.ch), CAB G 37.2

Contact Please contact Marc and Ulysse if you are interested in the project, and tell us a little bit about your mathematical background (e.g. attach a list of courses taken or a transcript of records).

References

[1] Karl Bringmann, Ralph Keusch, and Johannes Lengler. "Average distance in a general class of scale-free graphs with underlying geometry". In: *arXiv preprint arXiv:1602.05712* (2016).

[2] Karl Bringmann, Ralph Keusch, and Johannes Lengler. "Geometric Inhomogeneous Random Graphs". In: *Theoretical Computer Science* 760 (2015).

[3] Johannes Lengler and Lazar Todorovic. "Existence of small separators depends on geometry for geometric inhomogeneous random graphs". In: *arXiv preprint arXiv:1711.03814* (2017).